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AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

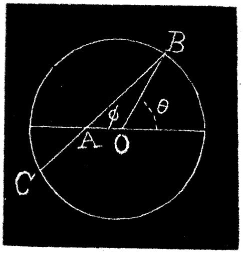
SOLUTIONS OF PROBLEMS.

43. **Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.**

In a circle whose radius is a, chords are drawn through a point distant b from the center. What is the average length of such chords, (1), if a chord is drawn from every point of the circumference, and (2), if they are drawn through the point at equal angular intervals?

Solution by the PROPOSER.

In the figure, let BC represent the chord passing through the point A whose distance from O is $OA=b$. Put $BC=x$, $\angle BOA=\theta$, $\angle BAO=\phi$, $A_1=$first average required, and $A_2=$second average required.



Then $x=2(a^2-b^2\sin^2\phi)^{\frac{1}{2}}$.

Hence, $A_1=1/\pi\int_0^\pi xd\theta$. From triangle AOB,

$a\sin(\theta+\phi)=b\sin\phi$. $\therefore$ $\theta+\phi=\sin^{-1}(b/a\sin\phi)$.

$$\therefore\ d\theta=-d\phi+\frac{b\cos\phi d\phi}{(a^2-b^2\sin^2\phi)}.$$

$$\therefore\ A_1=2/\pi\int_0^\pi[(a^2-b^2\sin^2\phi)^{\frac{1}{2}}-b\cos\phi]d\phi=\frac{4a}{\pi}\ E\left(\frac{b}{a},\ \tfrac{1}{2}\pi\right).$$

$$A_2=1/\pi\int_0^\pi xd\phi=\frac{4a}{\pi}\int_0^{\frac{1}{2}\pi}[1-(b^2/a^2)\sin^2\phi]^{\frac{1}{2}}\,d\phi=\frac{4a}{\pi}\ E\left(\frac{b}{a},\ \tfrac{1}{2}\pi\right).$$

$\therefore$ $A_1=A_2$. If $b=0$, $A_1=A_2=2a$. If $b=a$, $A_1=A_2=4a/\pi$.

This problem was also solved by G. B. M. Zerr. His solution will appear in the next issue.

44. **Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.**

What is the average length of all the chords that may be drawn from one extremity of the major axis of an ellipse if they are drawn at equal angular intervals?

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas; J. SCHEFFER, A. M., Hagerstown, Maryland, and the PROPOSER.

Let $r=$length, $e=$eccentricity of ellipse, $\theta=$angle the chord makes with major axis. Then

$$r=\frac{2a(1-e^2)\cos\theta}{1-e^2\cos^2\theta}.\quad \Delta=\text{average length}=\frac{\int_0^{\frac{1}{2}\pi}rd\theta}{\int_0^{\frac{1}{2}\pi}d\theta}.$$

$$\therefore\ \Delta=\frac{4a(1-e^2)}{\pi}\int_0^{\frac{1}{2}\pi}\frac{\cos\theta d\theta}{1-e^2\cos^2\theta}.\quad \therefore\ \Delta=\frac{4a\sqrt{1-e^2}}{\pi e}\tan^{-1}\frac{e}{\sqrt{1-e^2}}.$$

When $e=\theta$, $\Delta=\frac{4a}{\pi}$, since $\frac{1}{e}\tan^{-1}\frac{e}{\sqrt{1-e^2}}=1$.

MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Virginia. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

42. Proposed by E. B. ESCOTT, 6123 Ellis Avenue, Chicago, Illinois.

To find a triangle whose sides and median lines are commensurable.

Solution by J. W. TESCH, in "L'Intermediare des Mathematiciens" for October, 1896. Translated and adapted by J. M. COLAW, A. M., Monterey, Virginia.

Suppose the sides to be $2a$, $1+2b-b^2+\frac{1}{4}a^2$, $1-2b-b^2+\frac{1}{4}a^2$; then we will have $m_1=\pm(1+b^2-\frac{1}{4}a^2)$,

$$m_2{}^2=\tfrac{1}{2}[4a^2+(1-2b-b^2+\tfrac{1}{4}a^2)^2]-\tfrac{1}{4}(1+2b-b^2+\tfrac{1}{4}a^2)^2,$$

$$\text{or } m_2{}^2=\tfrac{1}{64}a^4+\tfrac{1}{8}(17-6b-b^2)a^2+\tfrac{1}{4}(1-12b+2b^2+12b^3+b^4).$$

In order that the second member may be a perfect square, it is necessary that $\frac{1}{64}(17-6b-b^2)^2=4\times\frac{1}{64}\times\frac{1}{4}(1-12b+2b^2+12b^3+b^4)$, whence $2b=3$.

Thus the sides become $2a$, $\frac{7}{4}+\frac{1}{4}a^2$, $-\frac{17}{4}+\frac{1}{4}a^2$, or

(1) $16a$, $2(a^2+7)$, $\pm2(a^2-17)$; $m_1=\pm2(a^2-13)$, $m_2=a^2+23$.

We will have $m_3{}^2=a^4+190a^2-191$. The values of a, which render the second number a perfect square, are 1, 3, 5, ... ; $m_3=0$, 40, 72, ... ; but none of these values satisfy (1) ; therefore, after the method of Euler, (*Vollständige Anleitung zur Algebra*, or the French translation by J.-G. Garnier, 2 vols., with the additions of Lagrange, Paris, 1807), it is necessary to proceed as follows :

By supposing $a=3+h$, we may write

$$(3+h)^4+190(3+h)^2-191=(40+ph+h^2)^2,$$

where p is a coefficient to be determined. Developing, we have

$$1248+244h+12h^2=80p+(80+p^2)h+2ph^2.$$